National Library of Canada
Canadian Theses Service
Ottawa, Canada
K1A 0N4

Bibliothèque nationale du Canada
Service des thèses canadiennes

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

NL-339 (r. 88/04) c

Canada

UNIVERSITY OF ALBERTA

REPRESENTATION OF COMPLEXITY: THE PRINCIPAL AXIS PRINCIPLE.

BY

DAVID L. HALL

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS.

DEPARTMENT OF PSYCHOLOGY

EDMONTON, ALBERTA

SPRING 1991

National Library of Canada
Bibliothèque nationale du Canada

Canadian Theses Service
Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN: 0-315-66638-2

Canada

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: David L. Hall

TITLE OF THESIS: Representation of Complexity: The Principal Axis Principle.

DEGREE: Master of Arts

YEAR THIS DEGREE GRANTED: 1991

PERMISSION IS HEREBY GRANTED TO THE UNIVERSITY OF ALBERTA LIBRARY TO REPRODUCE SINGLE COPIES OF THIS THESIS AND TO LEND OR SELL SUCH COPIES FOR PRIVATE, SCHOLARLY OR SCIENTIFIC RESEARCH PURPOSES ONLY.

THE AUTHOR RESERVES OTHER PUBLICATION RIGHTS, AND NEITHER THE THESIS NOR EXTENSIVE EXTRACTS FROM IT MAY BE PRINTED OR OTHERWISE REPRODUCED WITHOUT THE AUTHOR'S WRITTEN PERMISSION.

(SIGNED) David Hall

910-21 Ave NW

Calgary Alb

DATE: Dec 28 1990

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

THE UNDERSIGNED CERTIFY THAT THEY HAVE READ, AND RECOMMEND TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH FOR ACCEPTANCE, A THESIS ENTITLED REPRESENTATION OF COMPLEXITY: THE PRINCIPAL AXIS PRINCIPLE.

SUBMITTED BY DAVID L. HALL IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS.

(SIGNED) Alinda Friedman

Dr. Alinda Friedman
(Supervisor)

Dr. Peter Dixon

Roger W Toogood

Dr. Roger Toogood

DATE: Dec 28 1990

This work is dedicated to my friend, Susan.
Her support and encouragement made this possible.

Abstract

Two experiments using the mental rotation paradigm tested the hypothesis that the structure of the longest (principal) axis of an object would influence rotation performance. Stimuli were line drawings of three-dimensional shapes whose principal axes were either straight or bent. In Experiment 1, a minor axis (subarm) was also straight or bent. In Experiment 2, corresponding subarms on the two figures were either the same color as the principal axis or were colored differently to make them easier to find. In both experiments, higher intercepts and steeper slopes of the function relating reaction time to angular disparity were obtained when the principal axis was bent relative to when it was straight, and coloring the subarms did not eliminate this effect. These data are interpreted to mean that the structure of the principal axis is encoded at the top of the representation and the position of the subarm is encoded relative to the principal axis. Thus, the complexity of the principal axis exerts a large influence on the time to both encode and transform the representation during mental rotation.

Acknowledgement

This research was supported by a Natural Science and Engineering Research Council Grant to Dr. Alinda Friedman. I am grateful to Dr. Friedman for her excellent supervision and support throughout development of this project. I also thank Dr. Peter Dixon for advice and comments on previous drafts of this manuscript, Dr. Roger Toogood for serving as a member of my committee, and Dr. Gay Bisanz for acting as chair for the defence.

Table of Contents

Introduction 1

Background 4

A Theory of Hierarchical Representation . . 12

Testing the Contribution of the Principal Axis to Rotation Performance 14

Experiment 1 19

Method 20

Subjects 20

Stimuli and design 20

Procedure and Apparatus 22

Results 24

Discussion 30

Experiment 2 32

Method 32

Subjects 32

Stimuli and design 32

Procedure and apparatus 33

Results 33

Discussion 39

General Discussion 40

References 52

Appendix 75

List of Tables

Table 1. Intercepts in Experiment 1 58

Table 2. Slopes in Experiment 1 59

Table 3. Percent Error in Experiment 1 60

Table 4. Intercepts for Experiment 2 61

Table 5. Slopes in Experiment 2 62

Table 6. Percent Error in Experiment 2 63

List of Figures

Figure 1. Figures with a straight principal axis, showing straight subarm (top row) and bent subarm (bottom row). Figures on the right have been rotated 60 degrees around the oblique axis with respect to the figure on the left. 64

Figure 2. Figures with a bent principal axis, showing straight subarm (top row) and bent subarm (bottom row). Figures on the right have been rotated 60 degrees around the oblique axis with respect to the figure on the left. 65

Figure 3. Stimulus conditions used in Experiment 1 66

Figure 4. A figure with a straight principal axis and straight subarm shown in the 12 rotational positions that were used in these experiments. . 67

Figure 5. A figure with a bent principal axis and bent subarm shown in the 12 rotational positions that were used in these experiments. 68

Figure 6. Reaction time as a function of type of principal axis, type of subarm, and angular disparity for Experiment 1. 69

Figure 7. Percent error as a function of type of principal axis, type of subarm, and angular disparity for Experiment 1. 70

Figure 8. Design used in Experiment 2. For purpose of illustration, lines that were colored green in the study are outlined in bold. 71

Figure 9. Reaction time as a function of type of principal axis, color of subarm, and angular disparity for Experiment 2. 72

Figure 10. Percent error as a function of type of principal axis, color of subarm, and angular disparity for Experiment 2. 73

Figure 11. Cognitive process model for the tasks in Experiments 1 and 2. Arrows on solid lines indicate flow of processing. Dotted lines indicate some options after exiting Decision/Confirmation. 74

Representation of Complexity: The Principal Axis Principle.

Introduction

Mental rotation tasks involve imagining what objects would look like if they were rotated to different orientations or positions. Mental rotation can be required in a range of real world situations, from a laborer planning how best to load a truck, to an astronaut inside a space craft manipulating a robotic arm to repair a satellite.

An understanding of the factors that are involved in mental rotation tasks has both practical and theoretical significance. In the example of the astronaut, a practical question is how best to represent the three dimensional visual information on the two-dimensional screen of a control monitor, so that the astronaut watching the monitor can most effectively manipulate the arm. A related theoretical question is: How does the astronaut represent the three-dimensional structure of a shape in his mind, when that shape is portrayed on a two-dimensional display?

The research in this thesis investigated the mental representations and processes used when people encode, retrieve, and manipulate visuospatial

information. The findings are intended to contribute to the development of a theory for mental representation of three-dimensional objects. In two experiments, I investigated factors that contribute to the complexity of the structure of three-dimensional figures, and thereby make the task of mental rotation more difficult. An underlying assumption of this investigation is that factors that contribute to the complexity of the structure of a figure (figural complexity) will also contribute to the complexity of the mental representation (representational complexity) of that figure. In particular, I will propose that the mental representation of the structure of an object is hierarchical, that the principal axis (i.e., axis of elongation) of an object is an important part of the mental representation and is represented at the top of the hierarchy, and that the shape of the principal axis is a primary factor that determines figural and representational complexity.

Currently, there is not a generally accepted means of defining figural complexity. For example, a complex figure might have more parts, more vertices, extend in more directions, or be more difficult to label than a simple figure. Any one or more of these factors could possibly contribute to difficulty in a mental rotation

task. Further, a mental rotation task can vary in difficulty for reasons other than figural complexity. For example, depending on the position of a figure in a drawing, parts may be occluded, or the principal axis of the figure may be foreshortened. These, or other factors, may either make the structure of a stimulus difficult to determine, or may make the task of mentally manipulating the structure more difficult (for examples, see Biederman, 1987; Hinton, 1979; Marr & Nishihara, 1978). Thus, investigations regarding figural complexity also need to consider these potential factors.

In the following thesis I will first provide a general background for the mental rotation paradigm, and will introduce some findings regarding figural complexity in the mental rotation paradigm. I will then briefly discuss a theory that is congruent with these findings. A key principle in the theory is that the mental representation of the structure of an object is hierarchical, and that the principal axis of an object is represented at or near the top of the hierarchy. One prediction based on this notion is that the shape of the principal figural axis should have a greater influence on mental rotation performance than the shape of a minor axis (i.e., a subarm). I present

two experiments that explore this prediction. The experiments test whether the shape of the principal axis of a figure can be used as one operational definition of complexity for the mental rotation task.

Background

Shepard and Metzler (1971) showed that the time subjects require to decide whether two line drawings of three-dimensional objects are the same or different in shape increases linearly as the angular disparity between the two objects increases. The monotonic relation between angular disparity and reaction time for same/different judgments is robust, occurs with both two-dimensional and three-dimensional shapes, and has been replicated in numerous studies (See Shepard & Cooper, 1982, for review).

The slope and intercept derived from the function relating reaction time and angular disparity are thought to reflect different cognitive processes. The slope is thought to reflect the rate of a "mental rotation" process in which computations or transformations are performed on a mental representation of a depicted shape. The intercept is thought to measure non-rotation processes that include the rate of encoding (i.e., time to construct the mental representation), the comparison time once a

figure has been "mentally rotated," and the time to decide "same" or "different" (Cooper & Shepard, 1978; Just & Carpenter, 1985). Slopes and intercepts for trials in which the shapes are different are usually ignored, because it is not clear how to determine the rotational disparity of shapes that do not match.

Currently there are two general and opposing views about how to interpret mental rotation data. Some authors (Kosslyn, 1981; Shepard & Cooper, 1982; Paivio, 1975) suggest that mental representations of objects are holistic, and that they are functionally equivalent to pictorial representations. In such analog theories, the encoding, recall, and manipulation of visual information is performed in a manner that is different from the way in which verbal information is encoded, recalled, and manipulated. In these theories, mental rotation is functionally the same as a rigid rotation of a pictorial image.

Other authors (Hinton, 1979; Palmer, 1977; Pylyshyn, 1973; Reed, 1974) suggest that visual information is encoded in relational networks, or structural descriptions in the form of propositions, and that visual information is coded in a manner not different from the way that verbal information is coded. Further, in these propositional theories,

mental rotation is not functionally the same as an actual rotation of an image or picture.

Anderson (1978) suggests that both propositional and analog models often yield identical behavioral predictions, and evidence for either side of the debate is not conclusive. However, Pylyshyn (1979) argues that analog theories of representation predict that the time to rotate the representation should not be dependent on figural complexity. In this view, if complex figures do take longer to rotate, then a purely analog theory of representation is compromised.

It should be mentioned that Kosslyn and Schwartz (1977) suggested that it was possible for an analog theory to predict that images of complex figures might be more difficult to rotate, because such figures would be rotated in parts, and more complex figures presumably have more parts.

However, proponents of analog theories who suggest that some parts of an object may be mentally rotated separately from other parts of the object (for example, Bethell-Fox & Shepard, 1988; Kosslyn, 1981) need to account for the way that the original relations between the parts are preserved. For example, Kosslyn (1981, p54; see also Kosslyn, Roth & Mordkowitz, 1986) points out that underlying non-analog representations are

needed to define the location of parts, and outlines an analog model in which the information that specifies the location of one part relative to another is represented in a non-analog format. Such concessions enable a prediction of an effect of figure complexity on the rate of mental rotation, but also compromise an analog theory of object representation.

Proponents of propositional theories (Hinton, 1979; Palmer, 1977; Pylyshyn, 1979) predict that more complex figures will result in longer rotation times than simple figures. For example, Hinton (1979) suggests that a figure with a mental representation that is a complex structural description is more difficult to mentally rotate than a figure whose representation is a simple description. Hinton (1979; Hinton & Parsons, 1981) suggests that the structural description of a figure depends upon the number and kind of parts of the figure, as well as the positional relation of these parts to each other. Thus, the description of a complex figure might include many different relations between parts, and these relations might be difficult to maintain during a mental rotation (for further discussion of mental representation of structural descriptions see Marr, 1982; Palmer, 1977; Ullman, 1989).

It is clear that a theoretical explanation of the effect of figural complexity on mental rotation performance requires that the factors which determine complexity be identified. So far, empirical findings regarding the role of figure complexity have been mixed.

Several authors (Bethell-Fox & Shepard, 1988; Folk & Luce, 1987; Friedman & Hall, 1990; Friedman, Pilon, & Gabrys, 1990; Pylyshyn, 1979; Yuille & Steiger, 1982) have reported that, at least in some conditions, complex figures in rotation experiments produce steeper slopes and occasionally higher intercepts than simple figures. Other authors (Cooper & Podgorny, 1976; Cooper, 1975) have reported that complex figures do not affect the reaction time function. One reason for these conflicting results might lie in the distinction between the number of parts and number of parts relevant to the task. For example, there is some evidence that adding more parts to a figure does not result in increased rotation rates (slope) if those parts are redundant to the task of determining figure identity (Hochberg & Gellman, 1977; Yuille & Steiger, 1982).

Cooper and Podgorny (1976; see also Cooper, 1975) used two-dimensional polygons as stimuli for a rotation

task. As a manipulation of figure complexity, they varied the number of points on the polygons. Cooper and Podgorny (1976) reported that the number of points on the polygons did not affect subjects' rotation rates or accuracy, and concluded that the rate of mental rotation is not systematically related to the complexity of the internal representation. However, it has been argued (Folk & Luce, 1987; Hochberg & Gellman, 1977; Yuille & Steiger, 1982) that it might not have been necessary for subjects to rotate all parts of the figures used by Cooper (1975) and Cooper and Podgorny (1976). For example, though the polygons differed in number of points, each shape had one distinctive point that provided enough information for identification. Hochberg and Gellman (1977) reported that two-dimensional figures containing such readily identifiable "landmark" features were mentally rotated more quickly than figures in which such identity cues were absent, whereas the number of parts per se did not influence rotation time. In a similar vein, Corballis (1986, 1988) suggested that mental rotation is necessary when a task requires discrimination between left and right. In such cases, the parts of a figure needed to distinguish left from right might be those

relevant to rotation (see also, Hinton & Parsons, 1981; McMullen & Jolicoeur, 1990).

A clear distinction between redundant and relevant features is also pertinent to three-dimensional figures. For example, Yuille and Steiger (1982) pointed out that when a pair of standard Shepard-Metzler figures was rotated around the vertical axis, the figures could be discriminated as either the same shapes or mirror images by the bottom arm alone. Thus, when extra blocks were added to the main axis such that the shapes could still be discriminated by the bottom arm alone, response times for identity judgments were not influenced. However, when these extra blocks were added such that both the top and the bottom halves of the figures needed to be compared, slopes of the response time function increased.

Yuille and Steiger (1982) noted that only the parts of the figures that were necessary to discriminate shape identity were relevant to the solution of their rotation task. They suggested that people performed the mental rotation task by sequentially comparing parts of one figure with corresponding parts of the other figure (see also, Just and Carpenter, 1976, 1985). Thus, the effect of figural complexity on rotation performance was a

function of the number of parts that were necessary to determine figure identity.

Some recent studies (Friedman, et al. 1990; Friedman & Hall, 1990) compared simple and complex figures similar to those used by Shepard and Metzler (1971). The principal axis and axis of rotation of the figures was either the same (coincident condition) or different (noncoincident condition). The axes used were the vertical axis and an oblique axis not parallel to any of the three orthogonal planes in the XYZ coordinate system. The complex figures differed from the simple figures by a "kink" or bend in the principal axis and an extra bend on one subarm. Complex figures were more difficult to mentally rotate than simple figures, but only in the noncoincident conditions. Moreover, complexity defined in this way affected the performance of high and low spatial individuals similarly (Friedman & Hall, 1990). These studies (see also Parsons, 1987) illustrate that figural complexity should not be considered independently from the position of the principal and rotation axes of the figure. Although the complex figures used by Friedman et al. (1990; also Friedman & Hall, 1990) were different from the simple figures by virtue of a bend in the principal axis and a bend in the subarms,

Friedman et al. (1990; also Friedman & Hall 1990) suggested that the bend in the principal axis was more important in contributing to figural (and hence, representational) complexity. Experiment 1 directly tests this assumption.

In summary, from a review of the literature it appears that under some conditions (and definitions of complexity) more complex figures result in longer reaction times in the mental rotation paradigm. One parameter that contributes to figural and representational complexity could be the number of parts of a stimulus that are required to determine identity. A related and theoretically more interesting factor might be the relative location of these parts in terms of the overall structure of the stimulus. For example, a right angle bend in the principal axis might have a greater influence on rotation performance than a similar bend in a subarm, particularly if the shape of the principal axis and its position in space is an important component of the mental representation of an object's structure.

A Theory of Hierarchical Representation

Friedman, et al. (1990) suggested that the mental representation for the structure of objects is hierarchical, and that the principal axis and its

orientation with respect to the XYZ coordinate system are represented at the top level of the hierarchy. This suggestion is congruent with a theory proposed by Marr and Nishihara (1978; see also Marr, 1982) in which the principal axis of an object, and its orientation with respect to a frame of reference, are important for object recognition. In this thesis, I will assume that the principal axis of an object and its position in space is encoded with respect to a referent in a reference system which is extrinsic to the object (for example, coded with respect to the viewer, or with respect to the Y vertical axis, see also Friedman, 1990; Friedman & Pilon, 1990). Friedman et al. (1990; see also Marr and Nishihara, 1978) suggest that component parts of an object are encoded with respect to the principal axis. In such a representation, component parts (for example, subarms) of a figure would provide depth to its hierarchical representation.

One characteristic of the hierarchical representation that is suggested by Friedman et al. (1990) is that a change to one level of the hierarchy must be propagated down to other levels. For an object that has a principal axis, it is likely that changes to the position of that axis relative to its reference system would require changes to the top level of the

representation of the object in memory. Mental rotation performance (reaction time and errors) will be sensitive to such changes.

The notion that changes to the structure of the principal axis of a figure will make that figure more difficult to rotate is consistent with the results found by Friedman et al. (1990), and Friedman and Hall (1990). It is likely that such changes to the principal axis influence reaction times when the figure is rotated in a noncoincident condition, because in noncoincident rotation conditions the principal axis undergoes a substantial transformation with respect to an extrinsic reference. If changes to the structure of the principal axis do influence mental rotation performance, then an account of this influence should be explained in a description of the mental representation for object structure and in any cognitive process model for mental rotation.

Testing the Contribution of the Principal Axis to Rotation Performance

As mentioned earlier, the complex figures used by Friedman et al. (1990) and Friedman and Hall (1990) differed from the simple figures by both a bend in the principal axis and the addition of a bend to one subarm. Thus, it is not clear which of these additions

affected response times, or whether they both did. Therefore, in the first experiment the location of additional parts was manipulated independently, by placing them on one subarm, on the principal axis, or on both the principal axis and one subarm. Each figure was made of cube-like parts, and the shapes of the principal axis and subarms were either straight or bent, depending on the location of additional cubes.

We have previously found an effect of figure complexity only in noncoincident conditions (Friedman et al., 1990; Friedman & Hall, 1990); therefore, I used only noncoincident conditions in the present study. The principal axis of all figures was the vertical axis, and they were rotated around the oblique axis that was used by Friedman et al. (1990) and Friedman and Hall (1990). Rotations around this oblique axis were noncoincident with respect to the subarms as well as with respect to the principal axis. Thus, for these rotations, both the subarms and the principal axis change in position with respect to an extrinsic reference.

It is possible that foreshortening of the principal axis could make a rotation task difficult. However, because the principal axis of all figures had the same height, and because each figure type was

presented in the same set of positions, foreshortening of the principal axis was controlled across stimulus conditions.

Either a bend in the principal axis or the subarm could make the structure of the stimulus more difficult to parse, relative to when both of these stimulus parts are straight. If so, this difficulty would be reflected in a higher intercept when either the principal axis or the subarm is bent. However, if the shape of the principal axis has a greater influence on non-rotation processes than the shape of the subarm, then the intercept will reflect this greater influence.

It should be noted that, regardless of the shape of the principal axis, the parts of the stimuli that are necessary to determine overall shape identity are the same. As shown in Figures 1 and 2, the direction of the bottom 'foot' relative to that of the topmost subarm distinguishes all figures from their mirror image. For all figures, only one feature is necessary to determine identity, and thus, the number of relevant parts (i.e., the number of parts necessary to discriminate the shapes) is constant. Thus, according to the criterion suggested by Yuille and Steiger (1982; also Hochberg & Gellman, 1977), all figures have the same complexity.

On the other hand, stimuli with a bent principal axis (see Figure 2) can be distinguished by either the direction of the bottom 'foot' or the direction of the bend in the principal axis relative to the top subarm. Although the figures with a bent principal axis are distinguished from their mirror images by two features, either feature is sufficient to determine identity. This presents the possibility that the bent principal axis condition will actually result in faster slopes and intercepts than the straight principal axis condition, because the identity of bent principal axis figures can be distinguished via either one of two features, whereas figures with a straight principal axis can be distinguished by only one feature.

Only the hierarchical theory of representation predicts that rotation performance will be poorer with figures whose principal axis is bent than with figures whose principal axis is straight. Moreover, the theory can provide a basis to predict that the shape of the principal axis will influence the time required for encoding (intercept), and that the shape of the subarms and the shape of the principal axis might combine to influence the time required to process the mental representation (slope).

Assuming that a bent principal axis is more difficult to encode with respect to an external reference than a straight principal axis, encoding time should be longer when the principal axis is bent, relative to when the principal axis is straight. If the shape of the subarms is encoded with respect to an extrinsic frame of reference, then it is likely that encoding time will be longer for bent subarms than for straight subarms. However, I assume that the shape of the subarms is not encoded with respect to an external frame of reference; therefore, the intercept might not be sensitive to the shape of the subarms.

It was pointed out earlier that intercepts for trials in which the shapes are different are usually ignored. In the two experiments below, I included trials in which there was no rotational disparity; thus, I obtained data for actual intercepts on both "same" and "different" trials. Assuming that the encoding of a figure's structure precedes judgments about pair identity, then encoding time might not be sensitive to pair identity, but should be sensitive to stimulus condition. Thus, any factor that influences encoding time (for example, the shape of the principal axis) should be reflected in the reaction time for the actual intercept on both "same" and "different" trials.

Moreover, in the theory outlined above, the mental representation of an object's structure is hierarchical, such that the principal axis is represented at the top of the hierarchy and the subarms provide depth to the hierarchy. One principle of the hierarchy is that changes to the top of the representation must be propagated down to other levels. According to this view, the whole representation needs to change in this particular rotation task because the position of the principal axis changes relative to an external reference. The extent of the transformation will also be a function of the degree of rotation and the depth of the representation. It is likely that the process of transforming a representation (i.e., completing the mental rotation) becomes more difficult as the extent of the transformation increases. The time to make the transformation is indicated by the slope of the reaction time function. The difficulty in completing the transformation is indicated by reaction time and the number of errors. Assuming that changes to the representation are propagated down through the entire hierarchy, then the steepest slope and the most errors are likely to result when the principal figural axis and subarm are both bent.

Experiment 1

Method

Subjects. Six female and six male subjects were recruited from department research personnel and graduate students. They were paid five cents for every correct response. All had normal (or corrected to normal) vision. None had participated in a previous mental rotation experiment.

Stimuli and design. The shapes were constructed using a computer graphics package (Autocad 10, Autodesk Inc. 1988. See Appendix A for details of the display.). The shapes used for each condition are shown in Figure 3.

One example of each figure was labeled 'A', and its mirror image was labeled 'B'. Because this study also replicated previous work, the mirror image of each figure was constructed using the same plane of reflection as used by Friedman et al. (1990, see Figures 1 and 2, see also Appendix A).

Stimuli were initially constructed in a vertical position that is labeled 0 degrees. Eleven additional views of each A and B figure were then generated by rotating that figure in 30 degree increments around an oblique axis that was used by Friedman et al. (1990, see Appendix A). Thus, there were 12 views of each

figure, corresponding to 12 rotational positions. These are shown in Figures 4 and 5.

There were four sets of views, one set for each stimulus type (bent or straight principal axis by bent or straight subarm). From each set of 12 views, pairs were chosen to make all combinations that comprise each of six different relative angular disparities (i.e., one figure from 0 to 150 degrees apart from the other). There are a total of 72 pairs of figures (6 relative disparities X 12 pairs each). A given figure from each pair could be on either the right or the left, doubling the number of pairs to 144. In addition, each figure could be either the original figure (A), or its mirror-image (B). For a given angular disparity, there are 4 possible combinations of A and B figures: AA and BB "same" pairs, and AB or BA "different" pairs. Thus there are 576 (4 X 144) possible stimulus pairs for each of the experimental conditions.

The stimuli for a given subject were selected so that, across four subjects, all 576 pairs were seen exactly once, and each subject saw the identical pairs in terms of relative angular disparity, AA, BB, AB, or BA shapes, etc., in each condition. Stimuli were blocked so that in every 96 trials, there was one instance of each AA, BB, AB, BA pair at each of the 6

relative angular disparities for each of the 4 principal axis by subarm combinations. The stimuli were randomly presented within these blocks, with the constraint that no more than four consecutive pairs would require the same response. A new random order was generated for each subject.

Procedure and Apparatus. The subjects were seated in front of a Hewlett-Packard 1304a oscilloscope display with P15 phosphor. A plotting device developed by Finley (1985) was used to plot the figures on the oscilloscope during each session. Subjects placed their heads in a chin-rest with two laterally-placed head stops. The chin-rest was used to ensure that their heads were in a comfortable but stable vertical position throughout all the trials. A set of touchplates was on the table in front of the subjects. The touchplates were two wooden boards, each with a metal palm plate, a smaller metal finger plate and a (1.8 cm high) wooden strip beside the finger plate. Subjects placed one hand on each touchplate such that the heels of the palms rested on one metal plate and the index fingers rested on the wooden strip. Subjects responded by touching their index finger to the smaller metal plate. They used their preferred hand for "same" responses and the other hand for "different" responses.

A Zenith 159 computer timed the responses, and controlled the display devices.

Instructions for the task were read to the subjects. They were then given a short demonstration. In the demonstration, a pair of figures for one of the conditions was displayed on the screen. The left side figure was rotated by successive responses on the touchplates, until that figure was in the same orientation as the right side figure. This was repeated for eight pairs of figures: one pair of "same" and one pair of "different" figures for each of the stimulus conditions. The purpose of the demonstration was to make the concept of mental rotation explicit for the subjects, and to give them practice using the touchplates.

After the demonstration, subjects received 48 practice trials. Practice stimuli were randomly selected stimuli that were counterbalanced like the experimental trials that followed, but were only shown in 30, 90, and 120 degree disparities. Each trial began with a fixation cross in the center of the screen for 1 s. Then a beep was sounded, and .5 s later the cross disappeared and the figures were displayed. They remained on the screen until the subject responded. If the response was correct the figures disappeared and

the next trial began. If the response was incorrect, two beeps were sounded and the figures flashed off and then on again. They then remained on the screen for five additional seconds so that the subject could review his or her error. Practice trials were followed by two sessions which each had 288 experimental trials. The procedure was the same as for practice trials. Subjects received the same auditory feedback as the practice trials, but the figures did not return to the screen when the response was incorrect. After the first session, the subject was given a five minute rest, and then the second session was given. During practice and experimental trials the experimenter remained seated behind the subject. At the end of each set of trials, subjects were informed how many trials were correct and how much they had earned.

After the second session in the study, subjects were asked to describe the strategies they used to perform the task.

Results

Reaction times for correct responses on trials at zero degrees disparity (intercepts), and slopes of the function relating reaction time to angular disparity were each analyzed in a three-factor mixed design analysis of variance (ANOVA) in which within-subjects

factors were shape of the principal axis (straight, bent) and shape of the subarm (straight, bent) and the between-subjects factor was gender. The main effect of gender was not reliable for intercepts. Slopes were steeper for females than for males (21.5 vs. 13.7 ms/degree), $F(1,10) = 7.05$, $p < .05$. There was no reliable interaction between gender and the other factors for either measure, thus, all results reported below are from data that was collapsed over gender.

Subjects responded "same" and "different" with different hands, and so the "same" and "different" trials were analyzed separately. Intercepts for correct "same" trials were analyzed in a two factor analysis of variance (ANOVA) in which within-subjects factors were shape of the principal axis (bent, straight) and shape of the subarm (bent, straight). As predicted, the main effect of principal axis was reliable, $F(1,11) = 12.20$, $p < .01$. Figures with a bent principal axis produced a higher intercept than figures with a straight principal axis (1,358 vs. 1,190 ms). The shape of the subarm had no reliable influence on the intercept ($F < 1$), nor was the two-way interaction reliable for correct "same" trials ($F < 1$, see Table 1).

Intercepts for correct "different" trials were analyzed using the same design as that for "same" trials, and results showed a similar pattern (see Table 1). Intercepts were higher for the bent principal axis condition (3,274 ms) compared to the straight principal axis condition (1,677 ms), $F(1,11) = 32.56$, $p < .001$. The main effect of shape of the subarm was reliable, $F(1,11) = 20.74$, $p < .001$, but the interaction of Principal Axis X Subarm was also reliable, $F(1,11) = 38.23$, $p < .001$. A post-hoc analysis using a Newman-Keuls' test revealed that the intercept was higher for figures whose principal axis was bent relative to when the principal axis was straight, whether the subarm was straight or bent ($p < .05$, means are shown in the columns of Table 1). The intercept was highest (4,044 ms) when the principal axis and subarm were both bent ($p < .05$).

Slopes of the function relating reaction time and angular disparity on correct "same" trials were analyzed using the same design as that for the intercepts. Slopes were steeper in the bent principal axis condition than in the straight principal axis condition (19.2 vs. 15.8 ms per degree), $F(1,11) = 6.00$, $p < .05$. They were also steeper for figures with bent subarms than for figures with straight subarms

(21.5 vs. 13.4 ms per degree), $F(1,11) = 80.74$, $p < .001$, and the interaction between these factors was reliable $F(1,11) = 7.77$, $p < .05$ (see Table 2). Comparisons using orthogonal contrasts showed that when the principal axis was straight, the slope was steeper when the subarm was bent (17.9 ms per degree) relative to when the subarm was straight (13.7 ms per degree), $F(1,11) = 23.43$, $p < .01$. When the principal axis and subarm were both bent, the slope was steeper relative to when the principal axis was straight (25.2 vs 17.9 ms per degree), $F(1,11) = 8.28$, $p < .05$. When the subarm was straight, there was no reliable influence of shape of the principal axis on slope.

The errors were very high at 150 degrees disparity (41.7%) in conditions where both the principal axis and subarm were bent. This posed a risk that the reaction times at 150 degrees were not interpretable. Therefore, slopes were re-computed using reaction time data from only the first five angles of disparity (0, 30, 60, 90, and 120 degrees), and the new slopes were analyzed. There was no difference in results from the ANOVA, or for planned comparisons of the Principal Axis X Subarm interaction when the reaction time at 150 degrees was removed from the analysis.

The reaction times for correct "same" trials were analyzed in three-factor repeated measures design analysis of variance (ANOVA) in which within-subjects factors were shape of the principal axis (straight, bent), shape of the subarm (straight, bent), and angular disparity (0, 30, 60, 90, 120 or 150 degrees). These data are shown in Figure 6. The main effect of shape of the principal axis was reliable, $F(1,11) = 29.66$, $p < .001$. Figures with a bent principal axis produced longer reaction times than figures with a straight principal axis (2,982 vs. 2,475 ms). The effect of shape of the subarm on reaction time was reliable, $F(1,11) = 61.92$, $p < .001$. Figures with a bent subarm produced longer reaction times than figures with a straight subarm (3,073 vs. 2,384 ms). Reaction times increased as angular disparity increased, $F(1,11) = 55.67$, $p < .001$). All two-way interactions between these factors were reliable ($p < .05$), as was the three-way interaction of Principal Axis X Subarm X Angular Disparity, $F(5,55) = 3.22$ $p < .05$. Figures that had a bent principal axis and bent subarm produced the longest reaction times, particularly at the larger disparities.

Percent error for "same" trials was analyzed in an ANOVA of the same design as that for reaction times.

Data for percent error are shown in Figure 7. The main effects of Principal Axis and Subarm were reliable, $F(1,11) = 13.49, 35.79$ respectively, $p < .01$. Percent error increased with angular disparity, $F(1,11) = 35.5$, $p < .001$, and the two-way interactions of Principal Axis X Angular Disparity and Subarm X Angular Disparity were reliable ($F(5,55) = 5.91; 10.44$ respectively, $p < .01$). The interaction of Principal Axis X Subarm was also reliable, $F(1,11) = 7.67$, $p < .05$. The three-way interaction was not reliable for percent error, $F(5,55) = 1.36$, $p > .05$.

It was possible that the error responses on "same" and "different" trials were not equally distributed. To address this possibility, percent error for all trials was analyzed in a three factor repeated measures ANOVA in which within-subjects factors were trial type (same, different), shape of the principal axis (straight, bent) and shape of the subarm (straight, bent). Angular disparity was not a factor in this analysis because of the inclusion of "different" trials. Data are shown in Table 3. Neither the main effect of trial type, nor the interaction of Trial Type by Principal Axis was reliable. The two-way interaction of Trial Type by Subarm was reliable, $F(1,11) = 22.01$, $p < .001$, as was the three-way

interaction, $F(1,11) = 12.08$, $p < .01$. A post-hoc analysis using a Newman-Keuls' test confirmed that most errors were made when subjects responded "different" to pairs of figures that were actually the same shape, and whose principal axis and subarms were both bent (16.6%, $p < .05$, see Table 3). Thus, the most errors were made in a condition in which the task was expected to be the most difficult.

Discussion

The shape of the principal axis influenced both the slope and the intercept for the function relating reaction time to angular disparity, but the shape of the subarm primarily influenced the slope and the overall difficulty of the task.

I assumed that the time to encode a stimulus was manifested in the intercept. Thus, one interpretation of these data is that the shape of the principal axis influenced subjects' rate of encoding, but the shape of the subarm did not. However, there is another possibility. One strategy that subjects could use to perform the task is to look for corresponding ends of the two figures that comprised a pair (Just & Carpenter, 1976; Metzler & Shepard, 1974). When the principal axis is bent, corresponding ends might be harder to find than when principal axis is straight.

Thus, any influence on the intercept might be due to searching for corresponding ends rather than encoding the figures per se.

In summary, there are at least two possible interpretations of the results for the first experiment. One possibility is that the shape of the principal axis influences the time to encode the structure of that figure in memory, and that this time is manifested in the intercept. Another possibility is that the intercept is influenced by the time to find corresponding ends of the figures, and that the effect of the shape of the principal axis on intercepts is because the bend in the principal axis makes the ends hard to find.

One way that the two interpretations might be tested is to make the ends of the stimuli more easily identifiable. For example, if corresponding ends of two figures have the same color, and non-corresponding ends have a different color, then the task of identifying the corresponding ends should be facilitated. Thus, if the ends of the figures in Experiment 1 were harder to identify when the principal axis was bent, then coloring corresponding ends should reduce this difficulty. On the other hand, if figures with a bent principal axis were difficult because the

bent axis was more difficult to encode than the straight axis, then coloring the ends will not eliminate the difference in performance between the bent and straight-axis conditions.

Metzler and Shepard (1974, Experiment 2) placed same-colored dots in close proximity to corresponding ends of stimuli in a rotation study. They reported that color coding might have reduced the time to search for corresponding ends, but also pointed out that their comparison between presence and absence of color-coding was between experiments. Experiment 2 compared performance with pairs of figures whose corresponding bottom subarms were either colored or not colored.

Experiment 2

Method

Subjects. Six female and six male subjects were recruited from Department of Psychology research personnel and graduate students. They were paid five cents for every correct response. All had normal (or corrected to normal) vision. None had participated in a previous mental rotation experiment.

Stimuli and design. The figures were chosen from the set used in Experiment 1. All stimuli had a bent subarm. The shape of the principal axis was either straight, or bent. The stimuli were displayed as white

line drawings on a dark screen. The edges of the foot of the bottom subarm were either uncolored (white), or were colored a bright green (See Figure 8). Green was selected over the range of available colors because it seemed to be the most noticeable, and appeared to display with the same brightness as the rest of the figure. There were four combinations of principal axis (straight, bent) by color of subarm (colored, uncolored). The figures were the same size, were displayed at the same distance, and were rotated around the same oblique axis used in Experiment 1. Remaining within-subject and between-subject factors were counterbalanced in the same way as Experiment 1.

Procedure and apparatus. The same instructions and procedure were used as in Experiment 1 with the following exceptions: First, the researcher left the experiment room prior to the experimental trials. Second, each subject was instructed to initiate a given set of trials themselves by tapping a touch-plate one time. Third, to display the colored subarms, stimuli were shown using an NEC 3D Multisync monitor driven by a Paradise VGA video board rather than on the oscilloscope that was used in Experiment 1.

Results

Reaction times for correct "same" trials at zero degrees disparity (intercepts), and slopes of the function relating reaction time on correct "same" trials to angular disparity were each analyzed in a three-factor mixed design ANOVA in which within-subjects factors were shape of the principal axis (straight, bent) and color of the subarm (colored, uncolored) and the between-subjects factor was gender. For intercepts, neither the main effect of gender nor any interactions with gender were reliable. Slopes were steeper for females than for males (22.8 vs. 13.7 ms/degree), $F(1,10) = 5.84$, $p < .05$, but there was no reliable interaction between gender and the other factors on the measure of slope. Thus, all results reported below are from data that was collapsed over gender.

The intercepts from correct "same" trials were analyzed in a two factor ANOVA in which within-subjects factors were shape of the principal axis (straight, bent) and color of the subarm (colored, uncolored). As predicted, the main effect of shape of the principal axis was reliable, $F(1,11) = 30.37$, $p < .01$. Figures with a bent principal axis produced a higher intercept than figures with a straight principal axis (1,339 vs.

1,130 ms). In addition, figures with an uncolored subarm produced a higher intercept than figures with a colored subarm (1,290 vs. 1,179 ms), $F(1,11) = 5.92$, $p < .05$; however, the interaction between the two factors was not reliable. Thus, although color cues facilitated performance, this factor was independent of the influence of the shape of the principal axis. These data are shown in Table 4.

The intercepts from correct "different" trials were analyzed in an ANOVA of the same design as that for "same" trials. As predicted, figures with a bent principal axis produced a higher intercept than figures with a straight principal axis (2,824 vs. 1,532 ms), $F(1,11) = 17.81$, $p < .001$. Figures with an uncolored subarm produced a higher intercept than figures with a colored subarm (2,310 vs. 2,046 ms), $F(1,11) = 14.65$, $p < .01$. There was again no reliable interaction between the two factors on intercept (see Table 4).

Slopes of the function relating reaction time and angular disparity were analyzed in a two factor ANOVA in which within-subjects factors were shape of the principal axis (straight, bent) and color of the subarm (colored, uncolored). These data are shown in Table 5. Slopes were steeper when the principal axis was bent relative to when the principal axis was straight (20.6

vs. 15.9 ms/degree), $F(1,11) = 8.99$, $p < .05$. Slopes were also steeper for the uncolored subarm condition relative to the colored subarm condition (19.7 vs. 16.7 ms/degree), $F(1,11) = 10.30$, $p < .01$. The interaction between these factors was not reliable $F(1,11) < 1$. Planned comparisons revealed that in conditions where the subarm was colored, slopes were steeper when the principal axis was bent relative to when the principal axis was straight, (19.2 vs. 14.2 ms/degree), $F(1,11) = 14.23$, $p < .003$).

Again, the errors were high for figures at 150 degrees disparity in which the principal axis was bent and the subarm was uncolored (37.5%). As for Experiment 1, slopes were re-computed using reaction time data from only the first five angles of disparity. These data were then analyzed using ANOVA of the same design as the full data set. Slopes were steeper for the bent principal axis condition than the straight principal axis condition (21.0 vs. 15.6 ms/degree), $F(1,11) = 10.24$, $p < .01$. Using the trimmed data set, the effect of color proved not to be reliable, and the interaction between the factors did not attain reliability. For conditions in which the subarm was uncolored, a planned comparison indicated that the slopes were steeper when the principal axis was bent

relative to when the principal axis was straight (22.4 vs. 16.4 ms/degree) ms/degree, $F(1,11) = 9.51$, $p < .01$. For conditions in which the subarm was colored the slopes were also steeper when the principal axis was bent relative to when the principal axis was straight (19.6 vs. 14.9 ms/degree), but this difference only approached reliability in a planned comparison, $F(1,11) = 3.85$, $p < .076$.

Reaction time and percent error for correct "same" trials were analyzed in three-factor repeated measures design analyses of variance (ANOVA) in which within-subjects factors were shape of the principal axis (straight, bent), color of the subarms (colored, uncolored), and angular disparity (0, 30, 60, 90, 120 or 150 degrees). Shapes with a bent principal axis produced longer reaction times than those with a straight principal axis (3,009 vs. 2,302 ms), $F(1,11) = 51.20$, $p < .001$, and also produced more errors (12.2% vs. 7.8%), $F(1,11) = 10.56$, $p < .01$ (See Figures 9 and 10). Uncolored subarms produced longer reaction times than colored subarms (2,851 vs. 2,461 ms), $F(1,11) = 27.80$, $p < .001$, and also produced more errors (12.6% vs. 7.4%), $F(1,11) = 22.35$, $p < .001$.

Reaction time and errors increased as angular disparity increased (reaction time $F(1,11) = 53.31$;

percent error, $F(1,11) = 26.80$, $p < .001$ for both measures). The two way interaction of Principal Axis X Disparity was reliable for reaction time, $F(5,55) = 2.52$, $p < .05$, as was the Principal Axis X Color X Angular Disparity interaction, $F(5,55) = 3.22$, $p < .05$ (See Figure 9).

The interaction of Principal Axis X Angular Disparity was reliable for percent error, $F(5,55) = 3.13$, $p < .05$, as was the interaction of Color X Angular Disparity, $F(5,55) = 9.65$, $p < .001$ (See Figure 10). For percent error, the interaction of Principal Axis X Color was not reliable, and neither was the three way interaction of Principal Axis X Color X Disparity.

To address the possibility that error responses on "same" and "different" trials were not equally distributed, percent error for all trials was analyzed in a three factor repeated measures ANOVA in which within-subjects factors were trial type (same, different), shape of principal axis (straight, bent) and color of subarm (colored, uncolored). Errors were higher for "same" responses relative to "different" responses, $F(1,11) = 5.41$, $p < .04$. Errors were higher for figures whose principal axis was bent, compared to those whose axis was straight, $F(1,11) = 19.47$, $p <$

.001. The main effect of color was reliable, $F(1,11) = 10.74$, $p < .01$. The two-way interaction of Trial Type X Color approached reliability, $F(1,11) = 4.30$, $p < .062$. As can be seen from Table 6, subjects were most likely to incorrectly respond "different" to pairs of figures whose principal axis was bent and subarm was not colored.

Discussion

Once again the shape of the principal axis influenced both the slope and the intercept of the reaction time function. In addition, intercepts were higher in the bent principal axis condition than the straight principal axis condition for both "same" and "different" pairs. These data replicated those of Experiment 1, and support the notion that encoding time was influenced by the shape of the principal axis.

There are a number of possible reasons why intercepts were lower in the colored subarm condition relative to the uncolored subarm condition. For example, subjects might have used color to facilitate parsing the subarms from the principal axis, or to find corresponding ends of the stimuli. It is also possible that the color cues helped subjects keep track of the ends once the principal axis was encoded.

The time to transform the representation is expected to be dependent on the shape of the principal axis, and the data were concordant with this notion. The slope of the reaction time function was steeper for the bent principal axis condition than the straight principal axis condition. The color cues served to reduce slopes, but did not eliminate the effect of the shape of the principal axis on slope.

Coloring the subarms served to reduce errors overall; however, trials with the bent principal axis were more difficult than those with the straight principal axis, independent of color cues. Thus, although color cues might have facilitated performance overall, the effect of color did not eliminate the effect of the shape of the principal axis.

General Discussion

In two mental rotation experiments stimuli were used whose principal figural axis was either straight or bent. Results from both experiments were that encoding and rotation times were when the principal axis was bent. In Experiment 1, the intercept was influenced by the shape of the principal figural axis and not by the shape of the subarm. Experiment 2 showed that coloring corresponding ends of figures so that they would be easy to identify did not eliminate

the effect of the shape of the principal axis on time for either encoding or rotation.

Experiment 1 showed that once the structure of an object was represented, the time to process the representation in a "mental rotation" was influenced by both the shape of the principal axis and the subarm. The steepest slope of the reaction time function resulted when both the principal axis and the subarm of a figure were bent.

Results of these experiments support the notion that the shape of the principal axis is a factor that contributes to the complexity of the mental representation of an object's structure, and hence, to what constitutes figural complexity. Previous researchers have defined figural complexity in terms of the number of parts (for example, Bethell-Fox and Shepard, 1988; Cooper and Podgorny, 1976) or the number of parts required to determine identity (Hochberg and Gellman, 1977; Yuille and Steiger, 1982). However, in the present experiments, the number of parts that was required to solve the task was the same for all shapes. For all shapes, the direction of the bottom 'foot' relative to the top subarm could serve to distinguish a figure from its mirror image. The kinds of shapes differed by the presence or absence of a bend in the

principal axis or subarm, and these experiments have demonstrated that the particular location of bends in the structure influenced rotation performance. Specifically, the shape of the principal axis influenced the time required to encode a figure's structure as well as the time to complete computations on the representation. The shape of the subarms only influenced rotation times.

These results are consistent with a theory that the mental representation of an object's structure is hierarchical, such that the principal axis and its orientation are encoded at the top of the hierarchy (see also, Friedman et al. 1990; Marr & Nishihara, 1978). In this view, the subarms of a figure are represented at lower levels of the hierarchy relative to the principal axis, and thus, provide depth to the hierarchy. As discussed earlier, Friedman et al. (1990) suggested that mental rotation tasks in which the principal axis of the stimuli is different than the rotation axis (i.e., noncoincident conditions as in the present study) involve a transformation (or computations) to the top of the representation which must be propagated down the hierarchy. Thus, any change that is computed to the principal axis must also be propagated to the subarms. The extent of the

transformation that is necessary is a function of factors that include angular disparity and the complexity and depth of the representation. If the transformation is great, then the process of transformation will take longer and be more difficult to complete. The slope and error data in the present experiments were consistent with the notion that the mental representation of a figure with a bent principal axis was more difficult to transform than the representation of a figure whose principal axis was straight. To complete a transformation, changes to the top of the hierarchy must be propagated down through all levels of the representation; therefore, shapes with a bent principal axis and bent subarm were the most difficult, especially at large disparities.

Figure 11 depicts a cognitive process model that is consistent with the results of the present experiments, implements a hierarchical theory of representation, and includes processes for encoding a mental representation for the shapes used in the present experiments. The three stages of the process model are (a) encoding, (b) rotation and comparison, and (c) confirmation. These stages are similar to those proposed by Just and Carpenter (1976, 1985); however, an important difference between this model and

their (1985) model is that this model includes several encoding processes.

Encoding. There are three component stages to encoding. These components are (a) parse the figure, (b) encode principal axis relative to the Y axis, and (c) encode subarm(s) relative to the principal axis. In the first stage of encoding one figure is parsed into generalized cylinders along its component axes (Marr, 1982; refers to component axis as model axes, see also Marr & Nishihara, 1978). The cylinders align with the principal axis and subarms. For example, an L shape would have two component axes, (one vertical and one horizontal), that could be represented as two thin cylinders. Thus, all stimuli used in the present experiments will likely be parsed into three general cylinders: one for the principal axis, and one for each subarm. A shape will thus be initially parsed according to component axes for the principal axis and subarms, the principal axis and subarms will be represented as generalized cylinders, and then their specific shapes can be encoded (see Marr, 1982 for further discussion).

In the second stage of encoding a top is assigned to the principal axis (and thus, the figure) and the position of the top of the axis is referenced with

respect to the vertical Y axis. The time to compute the reference of the top of the axis with respect to the vertical axis is a function of the angular distance between the principal axis and the vertical Y axis (Friedman, 1990; Friedman & Pilon, 1990). The direction of rotation needed for the first shape to match the position of the second shape is computed. The shape of the principal axis is also encoded. The time to encode the shape of the principal axis increases as a function of the number of bends along that axis.

In the final stage of encoding the junction of each subarm is encoded relative to the adjoining part of the figure on the principal axis. It is likely that a join is labeled for a subarm in a similar manner that 'top' is assigned to the principal axis. Thus, while the principal axis is referenced to the vertical axis, the pos^Xxion of the subarms is encoded relative to the principal axis.

In this model, the shape of the principal axis can influence all of the stages of encoding, while the shape of the subarms might only influence the time to parse the structure. Thus, one reason why the shape of the subarms had such little influence on the intercept could be that the position of a subarm relative to the

principal axis is independent of the overall shape of the subarm. Information regarding the shape of the subarm is not required in this particular rotation task, and it is possible that the shape of the subarm is not encoded before the principal axis is rotated.

Rotation and comparison. The rotation and comparison processes are comprised of two stages. In the first stage, computations are completed for the rotation of the principal axis of the figure. Once these computations are completed, a subarm can be rotated and aligned with respect to the principal axis. Subarms are rotated (likely one at a time) in the second stage. During the second stage, computations might be needed to determine any effects of occlusion of a subarm by the principal axis as a result of rotation. In both stages it is likely that rotation is incremental, and that there is an ongoing comparison between the results from the computations and the appearance of the second stimulus (see also Just and Carpenter, 1976, 1985, for a similar proposal). Thus, the time to complete each stage would be a function of the extent of rotation, and the time to make the required comparisons. Rotation and comparison would stop when a decision regarding pair identity could be made.

Confirmation of decision. The subject decides whether the transformed representation of the first stimulus represents the same shape as the second stimulus. If a decision is "same", then the subject responds accordingly. If the decision is "different", or if the subject is not certain, then some processes might be repeated so that a decision can be made with confidence (see also, Just & Carpenter, 1985; Pellegrino & Kail, 1982).

Up until the computations for the rotation of the subarm are made, both "same" and "different" figures undergo the same processes. If the principal axis of a figure is straight, then the prediction from the model is that reaction time for "same" and "different" pairs would be the same until the subarm is rotated. If the principal axis is bent, and the figures are "different", then the discrepancy in shape might be noticed during the rotation and comparison stage for the principal axis, but not during encoding. Thus, the identity of a second stimulus might influence time for stages later than encoding, but not encoding time. Assuming that encoding time is a major component of the intercept, this interpretation is consistent with the data that intercepts for both "same" and "different" trials showed similar results.

In the model, the overall structure of a shape is encoded in stages. Subjects initially identify the principal axis and subarms so that the principal axis can be encoded, and later perform a more detailed analysis (and encoding) of structure as needed (see also, Marr, 1982; Marr & Nishihara, 1978). This interpretation could account for the benefit provided by color cues. A green subarm might be easier to discriminate from a white principal axis than a white subarm, but the principal axis must still be encoded. If the structure of the principal axis and the structure of the subarms are encoded independently, then the time to encode the shape of the principal axis would be independent of the time to encode either the shape or the color of the subarms. Congruent with this interpretation, the intercept for "same" trials in these experiments was not sensitive to an interaction of the shape of the principal axis with either the shape or color of the subarm.

The intercept did not vary with the shape of the subarms; therefore, I assume that the time to encode the relation of the subarms to the principal axis was not influenced by the shape^Xxf the subarm. However, if a unique relation to the principal axis is encoded for each subarm, then the number of subarms should

influence the intercept. Another possibility is that the shape of the subarms might have influenced encoding time if a different task was used which required information regarding the shape of the subarms. Encoding time is sensitive to the position of the principal axis relative to an extrinsic frame (Friedman, 1990); thus, if any effects of the number or shape of the subarms on encoding time could be found, such effects would likely interact with the position of the principal axis.

Computations of the transformation of the principal axis are made before the rotation of the subarms. It is likely that the results of the rotation of the principal axis must be maintained while computations are made for the subarms. One possibility is that computations (for example, those to determine rotation or occlusion) are more difficult when a figure has bends relative to when the figure is straight along all axes. Another possibility is that once the principal axis is rotated, the relation between a subarm and the principal axis is more difficult to maintain when the subarm contains bends. In either case, the rotation and comparison stage for a bent subarm might be more difficult compared to when the subarm is straight, and conditions in which the subarm

and the principal axis were both bent would be the most difficult. Either interpretation is consistent with the slope data from the present experiments.

Several points still need to be addressed. First, the current model does not explain how any computations are made. It is also assumed that certain computations will take time, but this assumption is difficult to assess without knowing what the actual computations might be. For example, those computations which reference the principal axis to the vertical axis will take longer as the distance from the principal axis and the vertical axis increases (Friedman & Pilon, 1990), but it is not yet clear what algorithm would behave in this manner.

Second, the theory does not address the nature of the representation for objects that have no principal axis. It is possible that the mental representation of any object is hierarchical; however, currently there is no comprehensive theory for the mental representation of all objects. Finally, I assume that a mental representation of an object's structure that would be used in a mental rotation task would share properties of a representation for the same object used in a different task (for example, object recognition).

However, the generality of the current theory for tasks other than mental rotation is not yet determined.

In conclusion, the results of the present study were congruent with the suggestion that an object's structure is represented as a structural description in a relational network (for example, see Hinton, 1979; Palmer, 1977; Pylyshyn, 1973; Reed, 1974). A particular relational network was suggested that is a hierarchical representation, such that the top of the hierarchy is a representation of the principal axis and its orientation with respect to an extrinsic reference (Friedman et al., 1990; see also, Marr & Nishihara, 1978). A cognitive process model was suggested that implemented the hierarchical representation, and is consistent with the results from two mental rotation experiments. These experiments have undoubtedly served to demonstrate that the shape of the principal axis should be included in an operational definition of figural complexity. Moreover, it is hoped that this study will contribute to a more complete understanding of the mental representation of three dimensional objects.

References

Anderson, J. R. (1978). Arguments concerning representations for mental imagery. Psychological Review, 85(4), 249-277.

Bethell-Fox, C. E., & Shepard, R. N. (1988). Mental rotation: Effects of stimulus complexity and familiarity. Journal of Experimental Psychology: Human Perception and Performance, 14(1), 12-23.

Biederman, I. (1987). Recognition-by-components: A theory of human image understanding. Psychological Review, 94(2), 115-147.

Cooper, L. A. (1975). Mental rotation of random two-dimensional shapes. Cognitive Psychology, 7, 20-43.

Cooper, L. A., & Podgorny, P. (1976). Mental transformations and visual comparison processes: Effects of complexity and similarity. Journal of Experimental Psychology: Human Perception and Performance, 2, 503-514.

Cooper, L. A., & Shepard R. N. (1978). Transformations on representations of objects in space. In E. C. Cartarette, & M. P. Friedman (Eds.), Handbook of Perception: Vol. 8. Space and Object Perception (pp. 75-176). New York: Academic Press.

Corballis, M. C. (1986). On imagined revolutions. In D. F. Marks (Ed.), Theories of Image Formation (pp. 151-168). New York: Brandon House.

Corballis, M. C. (1988). Recognition of disorientated shapes. Psychological Review, 59(1), 115-123.

Corballis, M. C., & Cullen, S. (1986). Decisions about the axes of disorientated shapes. Memory and Cognition, 14(1), 27-38.

Finley, G. (1985). A high-speed plotter for vision research. Vision Research, 25, 1993-1997.

Folk, M. D., & Luce, R. D. (1987). Effects of stimulus complexity on mental rotation rate of polygons. Journal of Experimental Psychology: Human Perception and Performance, 13(3), 395-404.

Friedman, A. (1990, July). The Importance of Being Upright: The Y Axis Advantage in Mental Rotation of 3D Objects. Paper presented at the Institute of Medical Psychology, University of Munich, Germany.

Friedman, A., & Hall, D. L. (1990, July). Representing and Manipulating Objects in Working Memory: Similarities Between High and Low Spatial Individuals. In C. D. Hardyck (Chair), Mental Image Generation, Rotation, and Interhemispheric Transfer: Perspectives on Subprocesses. Symposium conducted

at the meeting of the International Neurophysiological Society, Innsbruck, Austria.

Friedman, A., & Pilon, D. (1990). [The role of distance from upright in mental rotation]. Unpublished raw data.

Friedman, A., Pilon, D., & Gabrys, G. L. (1990). Cognitive coordinate systems for spatial working memory. Manuscript submitted for publication.

Hinton, G. (1979). Some demonstrations of the effects of structural descriptions in mental imagery. Cognitive Science, 3, 231-250.

Hinton, G., & Parsons (1981). Frames of reference and mental imagery. In A. D. Baddely and J. Long (Eds.), Attention and Performance (Vol. 9, 261-277). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hinton, G., & Parsons (1988). Scene-based and viewer-centered representations for comparing shapes. Cognition, 30, 1-35.

Hochberg, J., & Gellman L. (1977). The effect of landmark features on mental rotation times. Memory and Cognition, 5(1), 23-26.

Jolicoeur, P., & Kosslyn, S. M. (1983). Coordinate systems in the long-term memory representation of three-dimensional shapes. Cognitive Psychology, 15, 301-345.

Just, M. A., & Carpenter, P. A. (1976). Eye fixations and cognitive processes. Cognitive Psychology, 8, 440-480.

Just, M. A., & Carpenter, P. A. (1985). Cognitive coordinate systems: accounts of mental rotation and individual differences in spatial ability. Psychological Review, 92(2), 137-172.

Kosslyn, S. M. (1981). The medium and the message in mental imagery: A theory. Psychological Review, 88(1), 46-66.

Kosslyn, S. M., Roth, J. D., & Mordkowitz, E. (1986). Computational theories of image generation. In D. F. Marks (Ed.), Theories of Image Formation (pp. 131-149). New York: Brandon House.

Kosslyn, S. M., & Schwartz, S. P. (1977). A Simulation of visual imagery. Cognitive Science, 1, 265-295.

Marr, D. (1982). Vision. New York: W. H. Freeman.

Marr, D., & Nishihara, H. K., (1978). Representation and recognition of the spatial organization of three-dimensional shapes. Proceedings of the Royal Society of London, B, 200, 269-294.

McMullen, P. A., & Jolicoeur, P. (1990). The spatial frame of reference in object naming and discrimination of left-right reflections. Memory and Cognition, 18(1), 99-115.

Metzler, J., & Shepard, R. N. (1974). Transformational studies of the internal representation of three-dimensional objects. In R. L. Solso (Ed.), Theories in Cognitive Psychology: The Loyola Symposium (pp. 147-201). Potomac, MD: Lawrence Erlbaum Associates.

Paivio, A. (1975). Perceptual comparisons through the mind's eye. Memory and Cognition, 3(6), 635-647.

Palmer, S. E. (1977). Hierarchical structure in perceptual representation. Cognitive Psychology, 9, 441-474.

Parsons, L. M. (1987). Visual discrimination of abstract mirror-reflected three-dimensional objects at many orientations. Perception and Psychophysics, 42(1), 49-59.

Pellegrino, J. W., & Kail, R. Jr. (1982). Process analysis of spatial aptitude. In R. Sternberg (Ed.), Advances in the Psychology of Human Intelligence, (Vol. 1, pp. 311-365). Hillsdale, NJ: Lawrence Erlbaum Associates.

Pylyshyn, Z. W. (1973). What the mind's eye tells the mind's brain: A critique of mental imagery. Psychological Bulletin, 80(1), 1-24.

Pylyshyn, Z. W. (1979). The rate of "mental rotation" of images: A test of a holistic analogue hypothesis, Memory and Cognition, 7(1), 19-28.

Pylyshyn, Z. W. (1981). The imagery debate: Analogue media versus tacit knowledge. Psychological Review, 88(1), 16-45.

Reed, S. K. (1974). Structural descriptions and the limitations of visual images. Memory and Cognition, 2(2), 329-336.

Shepard, R. N., & Cooper, L. A. (1982). Mental images and their transformations. Cambridge: MIT Press.

Shepard, R. N., & Metzler, J. (1971). Mental rotation of three dimensional objects. Science, 171, 701-703.

Shepard, S., & Metzler, D. (1988). Mental rotation: Effects of dimensionality of objects and type of task. Journal of Experimental Psychology: Human Perception and Performance, 14(1), 3-11.

Ullman, S. (1989). Aligning pictorial descriptions: An approach to object recognition. Cognition, 32, 193-254.

Yuille, J. C., & Steiger, J. H. (1982). Nonholistic processing in mental rotation: Some suggestive evidence. Perception and Psychophysics, 31(3), 201-209.

Table 1

Intercepts in Experiment 1

Trial Type	Principal Axis	Subarm			
		Straight		Bent	
Same					
	Straight	1213	(265)	1168	(272)
	Bent	1342	(369)	1374	(358)
Different					
	Straight	1700	(439)	1654	(497)
	Bent	2504	(1098)	4044	(1727)

Note. Standard deviation is in parentheses. Means are based on 12 subjects.

Table 2

Slopes in Experiment 1

Principal Axis	Subarm	
	Straight	Bent
Straight	13.7	17.9
Bent	13.2	25.1

Table 3

Percent Error in Experiment 1

Trial Type	Principal Axis	Subarm	
		Straight	Bent
Same			
	Straight	4.7	8.5
	Bent	5.4	16.6
Different			
	Straight	3.0	5.3
	Bent	9.1	9.1

Table 4

Intercepts for Experiment 2[a]

Trial Type	Principal Axis	Subarm			
		Colored		Uncolored	
Same					
	Straight	1087	(298)	1174	(447)
	Bent	1271	(478)	1407	(476)
Different					
	Straight	1465	(306)	1599	(386)
	Bent	2626	(1326)	3022	(1229)

Note. Standard deviation is in parentheses. Means are based on 12 subjects.

Table 5

Slopes in Experiment 2

Principal Axis	Subarm	
	Colored	Uncolored
Straight	14.2	17.5
Bent	19.2	21.9

Table 6

Percent Error in Experiment 2

Trial Type	Principal Axis	Subarm	
		Colored	Uncolored
Same			
	Straight	6.4	9.3
	Bent	8.5	15.9
Different			
	Straight	6.2	6.7
	Bent	7.2	8.9

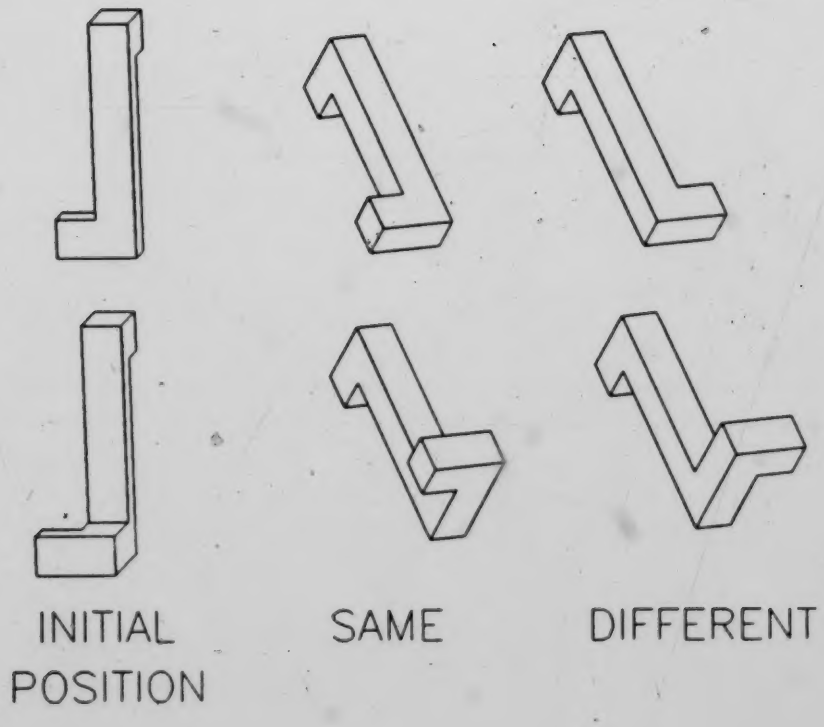

Figure 1. Figures with straight principal axis, showing straight subarm (top row) and bent subarm (bottom row). Figures on the right have been rotated 60 degrees around the oblique axis with respect to the figure on the left.

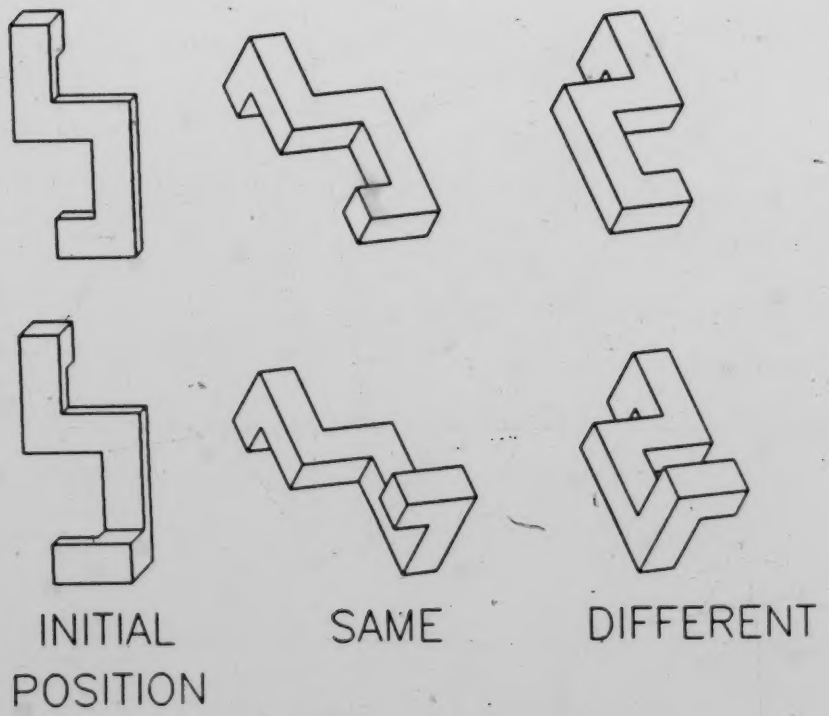

Figure 2. Figures with bent principal axis, showing straight subarm (top row) and bent subarm (bottom row). Figures on the right have been rotated 60 degrees around the oblique axis with respect to the figure on the left.

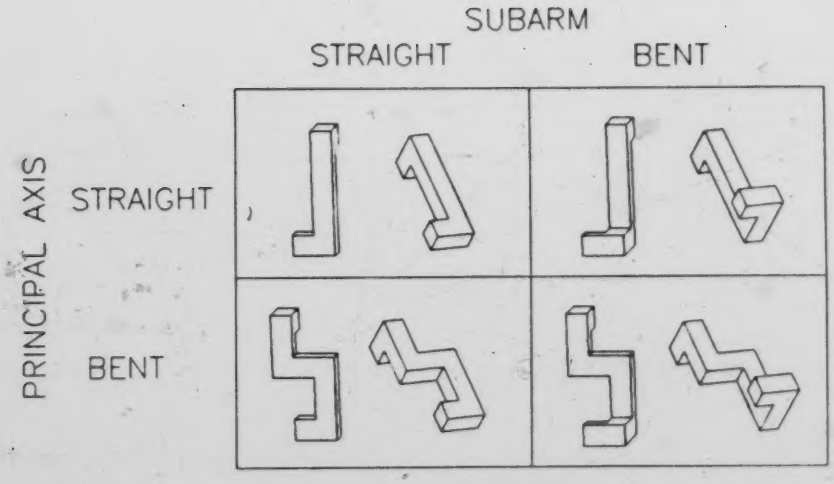

Figure 3. Stimulus conditions used in Experiment 1.

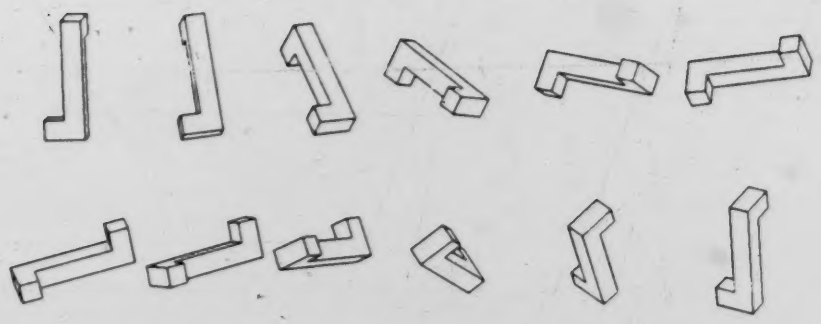

Figure 4. A figure with a straight principal axis and straight subarm shown in the 12 rotational positions that were used in these experiments.

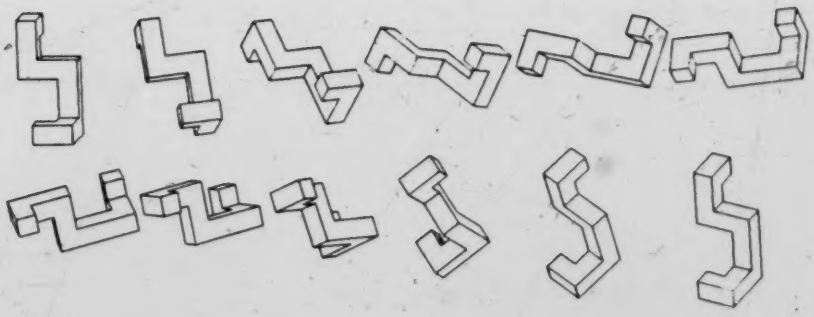

Figure 5. A figure with a bent principal axis and bent subarm shown in the 12 rotational positions that were used in these experiments.

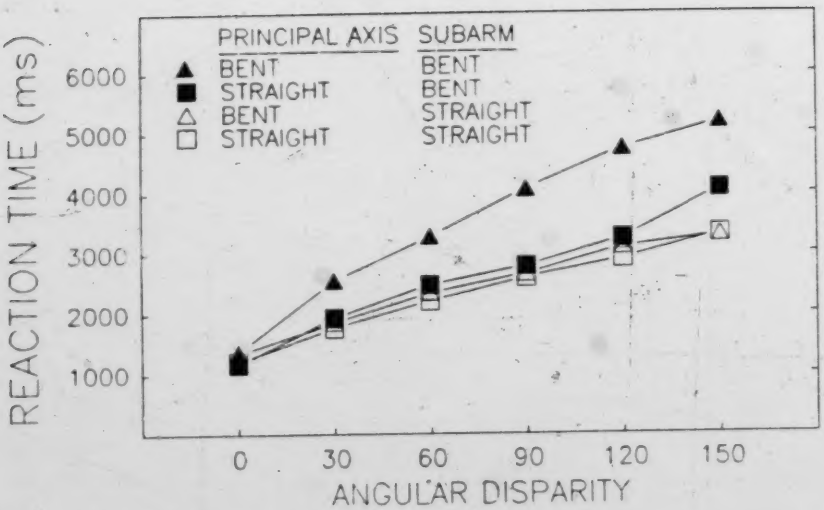

Figure 6. Reaction time as a function of type of principal axis, type of subarm, and angular disparity for Experiment 1.

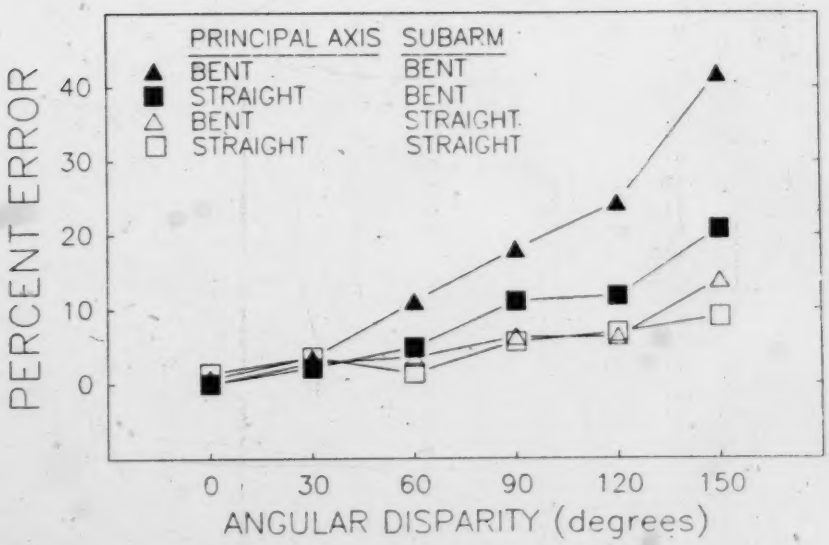

Figure 7. Percent error as a function of type of principal axis, type of subarm, and angular disparity for Experiment 1.

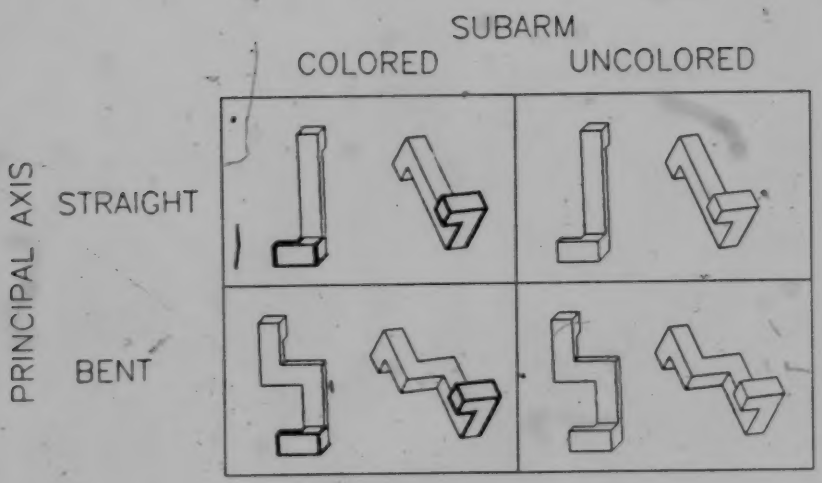

Figure 8. Design used in Experiment 2. For purpose of illustration, lines that were colored green in the study are outlined in bold.

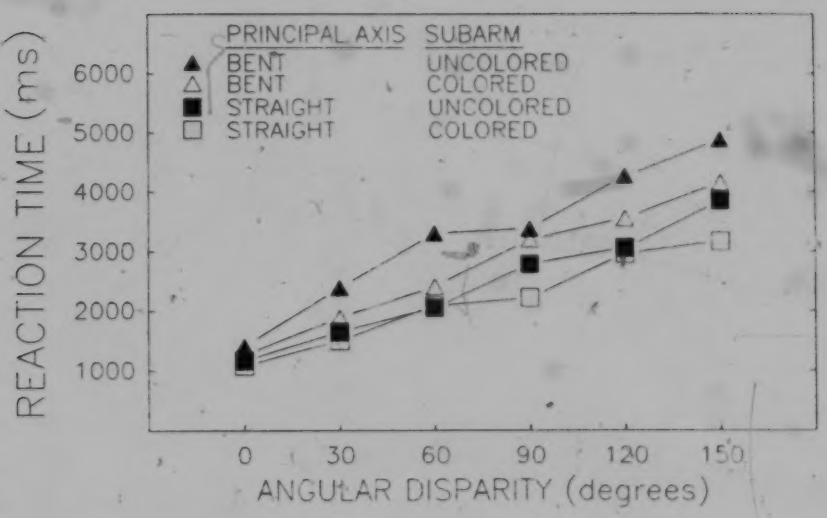

Figure 9. Reaction time as a function of type of principal axis, color of subarm, and angular disparity for Experiment 2.

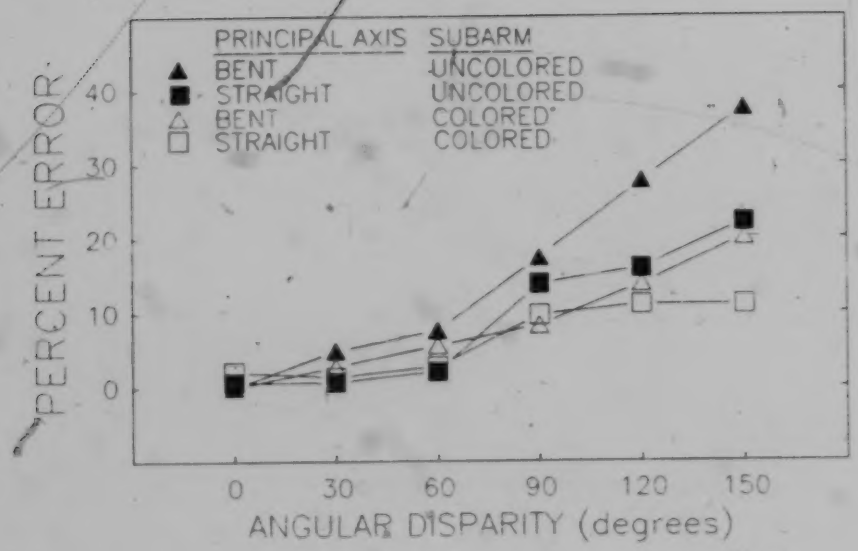

Figure 10. Percent error as a function of type of principal axis, color of subarm, and angular disparity for Experiment 2.

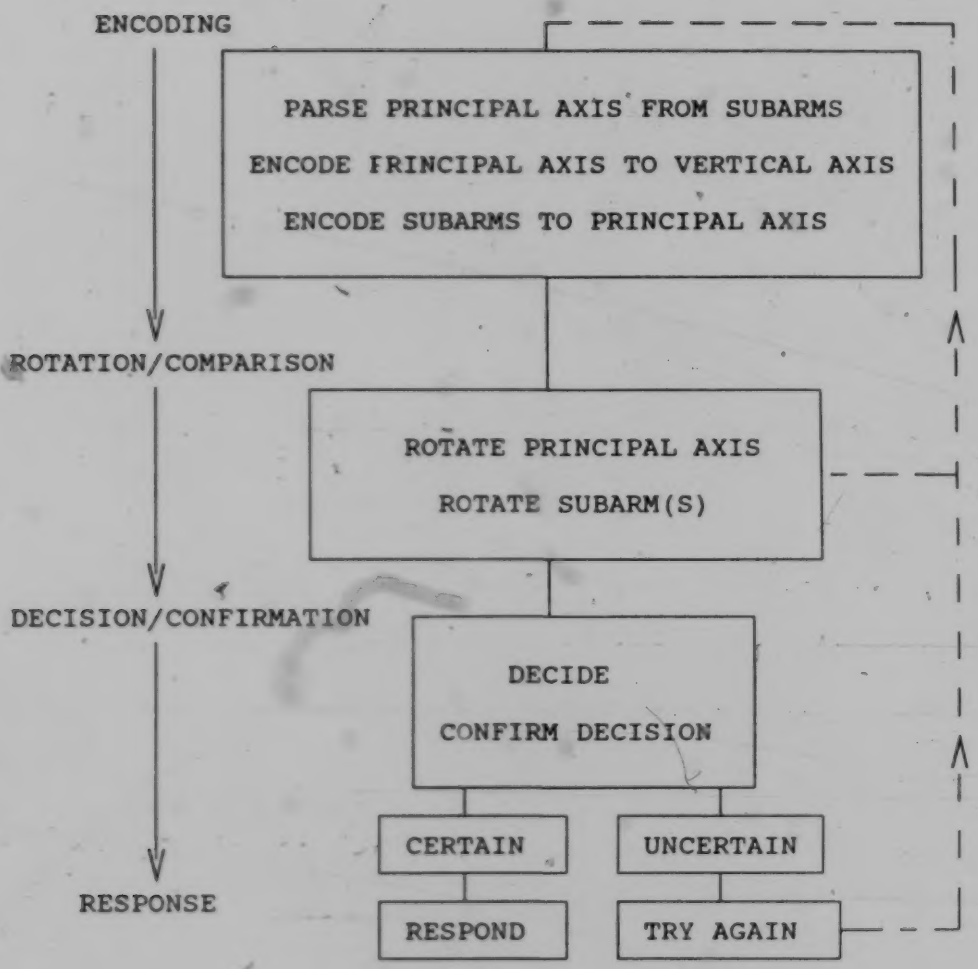

Figure 11. Cognitive process model for the tasks in Experiments 1 and 2. Arrows on solid lines indicate flow of processing. Dotted lines indicate some options after exiting Decision/Confirmation.

Appendix

Dimensions of Stimuli. Figures were drawn in unit measurements. All figures were drawn 6 units high, perimeters of subarms and main axes were 1 X 1 units. Thus, straight principal axes were 1 X 1 X 6 units and straight subarms were 1 X 1 X 1 units. The bend in either a principal axis or a subarm was made by adding a 1 X 1 X 1 unit piece that was orthogonal to the principal axis. Because the figures were shown in rotated positions, the actual height of each stimulus varied with the position of the stimulus. The height of a stimulus in it's initial position was 68mm in Experiment 1 (displayed on the HP oscilloscope), and 64mm in Experiment 2 (displayed on the NEC monitor).

Initial position. All figures were assigned an initial position which was labeled 0 degrees rotation. The initial position was achieved by rotating the figures toward the viewer 10 degrees about the horizontal (X) axis, and 10 degrees clockwise about the vertical (Y) axis. This position exposed more surface area relative to an orthographic projection (i.e., viewed straight on), which enabled the three dimensional structure of each figure to be seen (See Figures 1 and 2; also Friedman et al., 1990).

The rotation axis and PQR coordinate system. All figures were rotated about an oblique axis, labeled Q (See Figures 4 and 5). The coordinate system that was used to produce the stimulus rotations was designated PQR by Friedman et al. (1990). The PQR system was attained by three incremental rotations of a standard XYZ coordinate system. These are a 45 degree anti-clockwise rotation about the Y axis, a 35 degree clockwise rotation of the system about its X axis, and finally a 45 degree anti-clockwise rotation about its Z axis. The X, Y and Z axes of the rotated system are labeled P, Q, and R respectively. Thus, the Q axis is approximately 35 degrees from the X-Z plane, 12 degrees from the X-Y plane, and 52 degrees from the Y-Z plane (Friedman et al. 1990).

Display. Stimuli were displayed within 16.3 degrees visual angle, which was the visual angle of the screen in both experiments. The particular horizontal extent of a stimulus depends on its position. For all rotation positions in a given condition, the rotation axis intersected a single point (the rotation point) in the stimulus. This point was the same regardless of trial type (i.e., same or different). The distance between the rotation points of any pair of figures was constant for all trials. Experiment 1, the rotation

points were 102 mm apart, Experiment 2, the rotation points were 98 mm apart. The dimensions of the HP oscilloscope are approximately 275 X 212 mm. The dimensions of the NEC monitor are approximately 275 X 205 mm. The distance from the brow-bar of the headrest to the fixation cross in both experiments was 960 mm.